Reverberation Time Calculation Based on Room Modal Decay Using Wave Based Geometrical Acoustics & Modal Parameter Estimation

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INTRO & CONTENTS
INTRO

- Complex sound pressure Calculation
- Transfer Function
- Experimental Modal Analysis
- Modal RT
  - Resonant frequencies & Quality factors

1. Experimental Modal Analysis
2. Stabilization diagram
3. Modal RT
• The calculation of Modal RT is a 3-step process
• Below is Modal RT calculation flowchart for
  • symmetrical and
  • non symmetrical rooms

1. Summation complex pressure, \( p \)
   
   **TRANSFER FUNCTION Calculated**

2. EXPERIMENTAL MODAL ANALYSIS
   Variety of methods & packages
   
   a) Resonant frequencies
   b) Quality factors

3. MODAL RT current method

Wave Based Geometrical Acoustics (WBGA)
CONTENTS

1. What is Modal RT and how do we calculate it
2. What is Experimental Modal Analysis
3. What is Wave Based Geometrical Acoustics
4. Why do we need Modal RT
5. Conclusions
PLEASE NOTE:

• The content of the talk covers three big scientific fields.
• It is impossible to cover details for each of the fields.
• Therefore my aim is to convey concepts rather than cover in detail each field.

Let’s start backwards.
PART 1 a – WHAT IS MODAL RT?
PART 1 a – WHAT IS MODAL RT


“if the driving signal has its main frequency content far away from the resonance, there is hardly any reverberation, because the decay of energy can take place only near the resonance...”.
PART 1 a – WHAT IS MODAL RT

In 1963, Schultz produced the following RT graph, which demonstrates Heckl’s statement.

**Fig. 1.** “Reverberation Time” (T) of small broadcast studio.

PART 1a – WHAT IS MODAL RT

Calculation results of a recording studio where Modal RT is very important
PART 1a – WHAT IS MODAL RT

High resolution vs 1/3 octave band averaging
PART 1a – WHAT IS MODAL RT

1/3 octave band frequency analysis – Eyring vs Modal RT
PART 1 a – WHAT IS MODAL RT

Modal RT-Envelope & at resonant Frequencies
PART 1 b – MODAL RT CALCULATION
PART 1 b – MODAL RT CALCULATION

Reverberation Time (RT) is given by

\[ RT = \frac{6 \ln 10}{\delta}, \]

The **energy based** decay constant is

\[ \delta_{\text{average}} = \frac{cA}{8V} \]

\( A = \text{room sound absorption m}^2, \)

\( V = \text{room volume m}^3 \)

The **pressure based** decay constant is

\[ RT_r = \frac{6 \ln 10}{\delta_r}, \quad \delta_{\text{resonance}} = \frac{2\pi f_r}{Q} \]

\( f_r = \text{resonant frequency} \)

\( Q = \text{Quality factor} \)

Therefore, if the resonant frequencies and system damping can be extracted from TF, then Modal RT can be calculated.
PART 1 b – MODAL RT CALCULATION

However, it is a well known fact, that when driving a room close to a resonant frequency, nearby frequencies are partially excited too.

Fig. 24. Oscillograms illustrating beats in sound decay for different driving frequencies. Top and bottom curves are for driving frequency equal to a resonance frequency of the room, so only one mode is strongly excited. Middle curve is for an intermediate driving frequency, with two modes equally excited, showing the beats between the two natural frequencies as they damp out. Redrawn from reference K5.
PART 1 b – MODAL RT CALCULATION

Assuming a Gaussian distribution, then the Modal RT at one resonant frequency looks like the following figure.

From a previous paper by the same authors* 

\[ RT(f) = RT_r e^{-\frac{Q^2 \ln 2}{f_r^2}(f-f_r)^2} \]

RT Gaussian distribution 

\[ RT(f) = RT_r e^{-\frac{Q^2 \ln 2}{f_r^2}(f-f_r)^2} \]

Typical Gaussian distribution 

\[ F(x) \ = \ C e^{-\frac{1}{2\sigma^2}(x-x_m)^2} \]

* BEYOND SABINE: INVESTIGATING THE ACOUSTICAL PHENOMENON OF REVERBERATION USING ROOM MODAL DECAY, ICSV24, London
The Transfer Function (TF) is made up mostly of coupled modes thus making identification of resonant frequencies and their damping very difficult.

One needs to resort to specialized methods in order to extract modal parameters, methods found in Experimental Modal Analysis (EMA).
PART 2 – EXPERIMENTAL MODAL ANALYSIS (EMA)
Experimental Modal Analysis, is the study of the dynamical behaviour of systems, discrete or continuous. Safety requirements accelerated the development of accurate predictions and experimental analysis of physical system parameters.

In order to carry out EMA the following are required
- TF or IR of the system
- High resolution analysis

Currently, there are many methods in EMA with various degrees of success depending on the application. Below only concepts are conveyed.
From EMA, the resonant frequencies and damping are derived, allowing the calculation of Modal RT at each Resonant frequency.
Kenedy & Pancu (1947) developed a method to extract from lightly damped systems their modal parameters,

- Resonant Frequencies ($f_r$),
- their Amplitudes ($A$) and
- Damping ($Q$).

By plotting the Real and Imaginary parts of the Transfer Function, the method detects the parameters by fitting a circle on the Nyquist plot.
PART 2 – EXPERIMENTAL MODAL ANALYSIS (EMA)

1. First \( f_r \) (point B) is identified as the frequency at which there is maximum spacing on the arc of equal frequency increments. There we fit a circle.

2. Next the amplitude (line AB) is identified as the diameter of the circle.

3. Finally \( Q \) is determined by \( Q = (\omega_h - \omega_l)/\omega_c \)
   This is done by intersecting the diameter parallel to the real axis with the Nyquist plot. The intersection determines the half power width frequencies, \( \omega_h \) and \( \omega_l \) like in the peak amplitude method.
PART 2 – EXPERIMENTAL MODAL ANALYSIS (EMA)

The Algorithm of Mode Isolation (AMI) method

Extract from “Global and Multi-Input-Multi-Output (MIMO) Extensions of the Algorithm of Mode Isolation (AMI)”, PhD Dissertation by Matthew S. Allen
PART 2 – EXPERIMENTAL MODAL ANALYSIS (EMA)

In this paper the Least Squares Complex Exponential method was used

PART 3 – Wave Based Geometrical Acoustics (WBGA)
PART 3 – WAVE BASED GEOMETRICAL ACOUSTICS (WBGA)

What is Wave Based Geometrical Acoustics (WBGA)?

It is the method by which the calculation of acoustical fields takes into account the principle of superposition, using both amplitude and phase thus producing interference phenomena.

Below some of the pillars of the WBGA method:
PART 3 – WAVE BASED GEOMETRICAL ACOUSTICS (WBGA)

WBGA allows for:
• Edge diffractions & finite sized object reflections
• refraction and more.
• By using spherical wave angle depended extended reacting surface impedance,
  • we get complex pressures,
  • and phase,
  • in high resolution frequency analysis,
  • producing interference phenomena.
PART 3 – WAVE BASED GEOMETRICAL ACOUSTICS (WBGA)

The term Wave Based Geometrical Acoustics (WBGA), was first coined by Yiu Lam in his 2005 paper “Issues for computer modelling of room acoustics in non-concert hall settings”.

**Fig. 9** Comparison of wave based room acoustics models for predicting complex sound field in the standard listening room for the frequency range from 10 to 500 Hz with an assumed frequency independent absorptive admittance value of (0.2, 0.2i) for the floor.

**Fig. 10** 1/3 octave results of Fig. 9.
PART 4 – WHY DO WE NEED MODAL RT
If we examine a Waterfall Graph we see:
The resonant frequencies and their decay
PART 4 – WHY DO WE NEED MODAL RT

If we look at the same, top view, we are able to visualise Modal RT
PART 4 — WHY DO WE NEED MODAL RT

The typical method of calculating RTs
Examples, RT spectra in 63 and 1000 Hz 1/1 octave bands.

63 Hz 1/1 Oct.

1000 Hz 1/1 Oct.
PART 4 – WHY DO WE NEED MODAL RT

We get the typical RT octave band spectrum graph.
PART 4 – WHY DO WE NEED MODAL RT

If we superimpose the standard octave band RT over the Waterfall graph we see the following, as expected:
• Octave band RT overestimates RT and
• Ignores Modal RT which can reach very high values
PART 4 – WHY DO WE NEED MODAL RT

All the above calculations were derived from measured data provided by the International round-robin on auralisation https://rr.auralisation.net/
PART 4 – WHY DO WE NEED MODAL RT

The room where measurements were taken and used here in calculating Modal RT is shown below in a picture & SketchUp
PART 4 – WHY DO WE NEED MODAL RT

**STAGE 1:** Calculate the TF from the IR (provided by international project)
PART 4 – WHY DO WE NEED MODAL RT

**STAGE 2:** Extract Modal Parameters
PART 4 – WHY DO WE NEED MODAL RT

**STAGE 3:** Calculate Modal RT
PART 4 – WHY DO WE NEED MODAL RT

**MEASURED DATA:** TF vs Modal RT,
Observe that RT looks like the TF as it should.
Do not forget TF is at the beginning of a waterfall graph
PART 4 – WHY DO WE NEED MODAL RT

MEASURED DATA: Modal RT vs 1/3\textsuperscript{rd} Octave band averaging
Observe that RT NO longer look like the TF.
PART 4 – WHY DO WE NEED MODAL RT

Measured vs Calculated TF.
Calculation was too long and used: 9 orders of Reflection, 1 order of Diffraction

![Graph showing Measured vs Calculated TF](image.png)
PART 4 – WHY DO WE NEED MODAL RT

Measured vs Calculated Modal RT.
Calculation was too long and used: 9 orders of Reflection, 1 order of Diffraction
PART 5 – DISCUSSION & CONCLUSIONS
PART 5 – DISCUSSION & CONCLUSIONS

• Through out this presentation it is evident that there are modal components which yield high RTs.
• Yet standard calculation and measurements methods prefer to ignore them.
• The reason being that Modal RT is very difficult to calculate.
• However, averaging and the sound energy approach, provide the wrong interpretation of the processes taking place.
• Averaging gives the impression that all frequency components in a band share the same RT value.

DRAWBACKS OF THE ENERGY APPROACH (CALCULATIONS) & AVERAGING (MEASUREMENTS)

• **TF:** Even with the use of the Image source method, sound energy with or without averaging, can only provide amplitude and no phase
• **SOUND ABSORPTION:** The use of statistical absorption coefficients are energy based and averaged quantities which ignore angle dependent phase changes in sound absorption.
PART 5 – DISCUSSION & CONCLUSIONS

- **REFLECTION**: In calculations, are possible only from infinite surfaces, while averaging reduces or removes interference.
- **DIFFRACTION**: In calculations, no proper diffraction calculations are possible, while averaging reduces or removes interference.
- Energy based and averaged RT spectra do not correspond to TF.
- Energy based and averaged RT spectra do not correspond to sensation.

**BENEFITS OF WAVE BASED METHODS**
- Wave Based Calculations can account for all the points mentioned above. Their only drawback is that they are currently computationally expensive. With the advent of technology, that will no longer be the case.
CONCLUSIONS

• Room modal analysis reveals the true workings of sound in rooms.

• Wave Based Geometrical Acoustics has the power of carrying out modal calculations and in the future modal parameter estimation.

• As technology progresses, eventually the standard approach would be to use room modal analysis in all the useful frequency range, thus replacing the current approach of classical acoustics, its assumptions and limitations.
THANK YOU!

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