

Reverberation Time (RT) Calculation Based on Room Modal Decay and Using Wave Based Geometrical Acoustics (WBGA) and Modal Parameter Estimation

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Summary

In a previous paper, by the same authors, it was demonstrated that RT cannot be expressed as a continuous process in any domain; frequency, spatial or time. This paper is a continuation of the previous treatise, with the room being considered as a multi-resonant system. Its free vibrations, which aurally relate to reverberation, can be determined by the standard approach of the modal decay constant found in all linear systems irrespective of discipline. While in the previous paper, the methodology of implementing room modal RT was described for a cuboid room based on analytical expressions, this paper applies the same methodology for rooms of arbitrary shapes. With the use of Wave Based Geometrical Acoustics (WBGA) the transfer function (TF) of rooms of arbitrary shape is calculated. However, since resonances couple and overlap each other, the exact positions of modal frequencies become less evident. For this purpose, techniques of Vibration Modal Parameter Estimations used in Experimental Modal Analysis are applied here to detect resonant frequencies and their modal damping.

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1. Introduction

According to Heckl [1], during reverberation measurements, “if the driving signal has its main frequency content far away from the resonance, there is hardly any reverberation, because the decay of energy can take place only near the resonance... Strange results may occur when the driving frequency has its strongest component near but not exactly at the resonance;... in this case there may be beating phenomena Such phenomena may completely disturb an exponential decay”. Schroeder [2] has provided a remedy for the latter conditions. However, the current practice of calculating RT, is with a resolution of 1/1 and 1/3 octave bands. The process involves averaging, which inevitably gives the impression that the band RT applies evenly to all frequency band members. In contrast, calculating modal RT is quite a challenge, therefore the effort in this paper is to calculate modal based RT of single or coupled

resonances, which hopefully would yield better correlation between sensation and calculations. Once a concrete methodology is established and with the advent of computers, there would be no need to calculate room acoustical parameters based on statistical analysis. Wave based acoustics minimise the assumptions being made in the statistical approach of room acoustics, namely:

- a) That an isotropic and homogeneous acoustical field exists within a room,
- b) developed by sound reflections only,
- c) due to plane wave sound propagation.
- d) RT is calculated using the borrowed concept of the Mean Free Path from particle physics and optics,
- e) with the use of sound absorption coefficients and
- f) energy summation.

In a previous paper [3] the same authors have shown that Reverberation, being a modal phenomenon, is not a continuous process in any domain - frequency,

spatial or time and can be calculated using the room modal decay constant (related to the logarithmic decrement and loss factor in vibration) using wave based acoustics.

However, the calculation of the modal decay constant requires information of phase changes due to sound propagation discontinuities within a space, either due to reflections from impedance surfaces or impedance edge diffractions. While in the previous paper, the methodology of implementing room modal RT was described for a cuboid room based on analytical expressions, this paper applies the same methodology for rooms of arbitrary shapes. With the use of Wave Based Geometrical Acoustics (WBGA) [4] the transfer function (TF) of rooms of arbitrary shape is calculated. However, since resonances usually couple and overlap each other, the exact positions of modal frequencies become less evident. For this purpose, techniques of Vibration Modal Parameter Estimations used in Experimental Modal Analysis are applied here to detect resonant frequencies and their modal damping.

The paper is subdivided as follows: Section 2 is devoted to previous work done on similar methodologies while Section 3 introduces the methodology followed in this paper. Section 4 presents calculation results, while Section 5 discusses the findings of the results. The paper concludes with Section 6.

2. Previous work

2.1. Introduction

Rooms represent, in terms of modal analysis, continuous systems with an infinite number of modes and their associated natural frequencies. Room modal analysis can be carried out analytically in symmetrical rooms, such as cuboids. The aim of the paper is to propose a method to carry out modal analysis of a room of arbitrary shape using WBGA in conjunction with Modal Parameter Estimations extensively used in the field of Vibration. Wave Based Geometrical Acoustics is the Geometrical Acoustics method by which the calculation of acoustical fields uses complex pressure summation producing interference phenomena.

As far as the authors are aware, for asymmetric rooms, only numerical methods (for example FEM, BEM) can be used in calculating room modal parameters such as natural frequencies, their amplitude and damping. Nevertheless, there is a lot

of work in room acoustics which could be used towards extracting modal parameters such as the work found in references [5, 6, 7]. The main difficulty however, remains that resonances couple and overlap each other. This is where the Inverse Problem approach [8] applies which has been the focus in the field of experimental modal analysis.

2.2. Experimental Modal Analysis

Modal Analysis, concerns itself with the study of the dynamic behaviour of a system independent of the input on the system [9]. The dynamical behaviour of continuous systems, have been the object of research in the field of vibration especially since the advent and growth of aviation and space industries, to mention a few, where accurate predictions and experimental analysis of physical system parameters are of utmost importance for structures subjected to unpredictable loads [10].

While the behavior of dynamic systems can be described by differential equations, analytical models are often intractable to solve for many real world systems without resorting to numerical methods. An alternative method of describing dynamic systems is using their modal parameters, such as the natural frequencies, damping ratios and residuals of the principal modal components of the system [11]. These parameters have the advantage that they can also be obtained experimentally. There are many methods by which to extract physical parameters such as vibration modes and structural damping. However, none of them offer full proof parameter identification either from measurements or simulations, due to the complexity of continuous systems [8, 10].

This is because the main challenge of extracting the modal parameters from the experimental (or simulated) transfer function of a system is that of an Inverse Problem approach. Whereas in an analytical model the response of a system is calculated from the known matrices of the system, modal analysis extracts these matrices from the transfer function of the system. This is particularly difficult when resonances overlap each other, thus the nature of vibrational or acoustical responses become unclear. Investigating the methodologies used in Vibration [11], one is able to realise that there is extensive research which over the years has produced various modal parameter extraction methodologies. The first and most obvious way to estimate modal parameters is with the Peak-Amplitude method [10], which involves identifying the peaks of a transfer function to identify the natural frequencies

of a system and using the sharpness of the peaks to identify damping. The main flaw of this method is that the peak of the transfer function does not correspond to the true natural frequency of a system. Furthermore this method is not capable of distinguishing two modes whose natural frequencies are so close to each other that their peaks are indistinguishable. In 1947 Kennedy and Pancu [12] proposed an improved circle fit method where the phase of the system is also included in the analysis using the Nyquist plot of the transfer function. Their method leads to a more accurate estimation of the natural frequencies of modes when there is modal overlap. Damping can also be estimated directly for each resonance.

These two methods are limited to single degree of freedom systems where the resonant frequencies are evenly spaced apart from each other. Due to these limitations, new methods were developed based on multiple-point excitations. Although they were theoretically able to distinguish strongly coupled modes, they still had other practical limitations. The method of Traill-Nash [13] requires that the number of exciters used is equal to the degrees of freedom of the system, which for continuous systems is infinite and even though in a limited frequency range there is a finite number of modes the number is still not known beforehand. Asher's method [14] overcame these difficulties by successively approximating the zeros of the determinants of several response matrices. However, due to the practical limitations of the epoch, the method was not able to be sufficiently verified experimentally. As computational power and signal processing capabilities improved over the next decades more sophisticated algorithms developed which could utilise entire datasets of Frequency Response Functions from multiple receiver and exciter points of structure. In the 1970s the Least Square Complex Exponential (LSCE) algorithm was developed which uses a least-squares method to identify the system eigenvalues and then uses those values to estimate in turn the residues which allow the user to study stabilisation diagrams [15]. A more in depth review of the various new methods that developed in the last few decades can be found here [16].

One additional difficulty when using more recent methods is the correct estimation of the modal order of the system (the number of modes within the frequency range). A recent promising method is the Algorithm of Mode Isolation (AMI) [17, 18]. With this method, instead of trying to identify all of the modal components simultaneously, each mode is

calculated individually and estimated using a Single Degree of Freedom (SDOF) method. Then by subtracting that particular modal component from the transfer function, makes the rest of the modes more dominant. Once this phase of the algorithm is complete, i.e., until all remaining dominant modes are sequentially subtracted until the residual of the original TF is just noise, an assessment of the modal order of the system can be made available. The next phase of the procedure estimates the parameters again using a Single Input Multi Output (SIMO) algorithm. Since the first phase of the algorithm relies on SDOF method, the algorithm has the potential to also provide a semi-automatic method of also establishing the modal order of the system.

3. Methodology Used

In order to obtain the modal RT of a room of arbitrary shape, the modal parameters of the transfer function of the system need to be identified. For this purpose the commercial software Olive Tree Lab-Suite (OTL-Suite) was used [19]. OTL-Suite incorporates Wave Based Geometrical Acoustics (WBGA) [4] in its calculation engine, and uses spherical wave propagation, complex pressure summation, extended reacting surface impedances, high order diffractions and reflections, Fresnel Zone correction, thus preserving phase information between different wave fronts. It also allows for the calculation of frequency responses at arbitrary frequency ranges and resolutions, an essential capability for this study as it concerns itself with the detailed analysis of the low frequency behavior of rooms.

3.1. Calculating the Transfer Function of a Room

The transfer function of a room of general shape was calculated using WBGA. The first step of WBGA is to use the Image Source Method (ISM) to find the sound paths in a room which propagate from a point source to a receiver. From the sound paths the required parameters for calculating the total complex-pressure based frequency response (p_t) are found. These are the path length (R_i), the reflection order (o_i) and the angle-dependent product of attenuation factors, Q_j , D_l and α_{R_i} . These parameters are then used to evaluate the following equation shown below:

$$p_t = \sum_{i=1}^n p_o \frac{e^{ikR_i}}{R_i} \prod_{j=1}^m Q_j \prod_{l=1}^s D_l \alpha_{R_i} \quad (1)$$

where Q_j corresponds to the j th reflection factor of m total reflection factors of a path, D_j corresponds to the j th diffraction factor of s total diffraction factors of a path while α_{R_i} corresponds to the atmospheric attenuation of path i .

Because the evaluation of the transfer function is complex pressure based, WBGA is capable of accommodating extended reacting impedance surfaces, spherical wave propagation, interference phenomena between paths as well as diffractions or scattering from impedance edges.

3.2. Estimating the Modal Parameters

One popular such identification method is the Least-Square Complex Exponential method [15]. The LSCE method is applicable to Multi Degree Of Freedom (MDOF) systems and it calculates the system poles λ_r of equation 1 in the time domain by making use of the impulse response functions (IRFs) obtained from Frequency Response Functions (FRFs) by an inverse Fourier transform (IFFT). In partial fraction form, the Transfer Function (TF), $H(\omega)$, of a general system is given by equation 2:

$$H(\omega) = \sum_{r=1}^n \left(\frac{A_r}{i\omega - \lambda_r} + \frac{\bar{A}_r}{i\omega - \bar{\lambda}_r} \right) \quad (2)$$

where a bar denotes the complex conjugate, $A_{rs}(r)$ denotes the residue while λ_r denotes the system poles defined by equation 3:

$$\lambda_r = \zeta_r \omega_r + i\omega_r(1 - \zeta_r^2)^{1/2} \quad (3)$$

where ζ_r denotes the damping ratio of the r th mode. The LSCE algorithm used is the 'lsce.m' function available in the open source MatLab toolbox EasyMOD [20, 21]. The 'lsce.m' function in the EasyMOD toolbox allows the user to control the number of iterations used, as well as the tolerance for the estimated values of the natural frequency and the loss factor η , which is related to the damping ratio ζ via the relation $\zeta = \eta/2$.

The algorithm starts by detecting the peaks of the transfer function, p_{TF} . A set of database points in the vicinity of each peak are selected and are processed by the LSCE algorithm. The selection of the data points in the vicinity of the peak is done as follows:

$$p_{tf} > \varepsilon * \max(|p_{tf}|) \quad (4)$$

The selection of a suitable parameter for ε is important for obtaining a good estimation of the damping factor. The closer the value of ε is to 1, the more of the TF is being trimmed which may result in a less accurate estimation of the modal parameters. On the other hand when the peaks of two distinct modes are very close to each other a high value of ε may be necessary in order to exclude

the contribution of the second mode from the data selection. Modes close to each other lead to a high estimation of the damping factor. The LSCE method then returns candidate values of the eigenvalue that best fit the selected data points.

3.3. Plotting the Final RT Curve

Once the natural frequencies and damping of the system are known, it becomes possible to plot the RTs of individual resonant modes of the system which can then be combined to the total RT of the system, using the decay constant of a natural frequency δ_r :

$$\delta_r = 2\zeta_r \omega_r, \quad RT_r = \frac{3 \ln 10}{\delta_r} \quad (5)$$

Whenever damping is present in a system, the frequencies near a natural frequency are also excited to some extent, and the amount by which these frequencies are excited by, is related to the Quality Factor (Q_r) of the mode:

$$Q_r = \frac{\omega_r}{\Delta\omega} = \frac{f_r}{\Delta f} = \frac{\pi f_r}{\delta_r} \quad (6)$$

The shape of the peak may be modelled by the normal distribution described in the equation 7:

$$F(f) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(f-f_{mean})^2}{2\sigma^2}} \quad (7)$$

where σ is the standard deviation. In this case the natural frequency corresponds to the mean of the distribution while Δf corresponds to the Full Width Half Maximum (FWHM) of the distribution which is related to the standard deviation:

$$\Delta f = FWHM = \frac{\delta_r}{\pi} = 2\sqrt{2 \ln 2} \sigma \quad (8)$$

Substituting for f_{mean} and σ in equation 8 above we arrive at:

$$RT(f) = RT_r e^{-\frac{(\pi)^2 2 \ln 2 (f-f_n)^2}{\delta_r^2}} \quad (9)$$

where RT_r is used as the scaling factor of the distribution. RT curves of individual modes can then be plotted individually. The envelope of the response of the spectrum is obtained by taking the maximum RT value of all the individual spectra at each frequency f . More details on the methodology for plotting the RT curves of individual modes can be found in [3].

4. Results

4.1. Cuboid Room with uniform acoustical impedance

For comparison purposes with analytical methods (not done here), the Modal Parameter Estimation technique was applied to a cuboid room with dimensions of 5x4x3 m³. Both the source and receiver were placed in opposite corners of the

room, where all modes would theoretically be excited.

Different boundary conditions were tried on the walls of room. First a uniform impedance was assigned to all the walls of the room which was calculated using the Delany and Bazley 1-parameter model [22]. First a flow resistivity of 20,000 kPasm⁻² (hard) was used and then a flow resistivity of 200 kPasm⁻² (soft). For the Image Source Method 16 orders of reflection were used while the frequency range spanned from 1 to 100 Hz at 0.1 Hz intervals. Each WPGA calculation lasted for about an hour. For the LSCE algorithm variations of the tolerance of the loss factor didn't have any significant impact on the estimated values of η . However, the tolerance of the natural frequency did affect the number of candidate modes found. In both cases a tolerance of 1% was used. It was found that an iteration number of 25 was enough to lead to stabilisation in the estimation process (for explanation of terms see [15, 20]).

Figure 1 on the Left, shows the RT curves of the individual modal components (thin curves) along with their envelope (thick curves) for the room with flow resistivity of 20,000 kPasm⁻² (red) and 200 kPasm⁻² (blue). Figure 1 on the Right, compares the TF of the two cases. The effective statistical absorption coefficient corresponding to the "soft" DB impedance is given in Figure 2 (R).

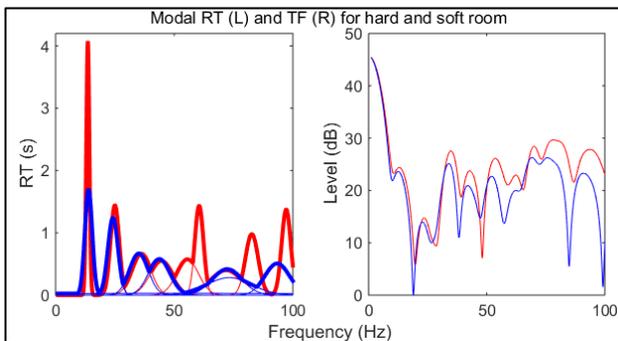


Figure 1. (L) Comparison of the RT spectra of the envelopes and individual modal components of the cuboid room, (red & blue curves for hard & soft materials respectively). (R) Comparison of the levels of the TF of the cuboid room (red & blue curves for hard & soft materials respectively).

4.2. Cuboid Room with non-uniform acoustical impedance

In the second case each wall was assigned a different acoustical extended reacting impedance based on the model of Allard and Johnson (AJ) [23]. Figure 2 on the left, below shows the equivalent

statistical absorption coefficients of the AJ model impedance.

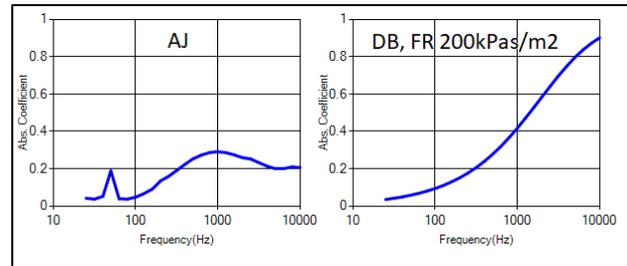


Figure 2. Equivalent statistical absorption coefficients of the 5x4x3 m³ cuboid room. (L) AJ (R) DB.

For comparison purposes, calculations were carried out once with proper WPGA, while in the second case using the Classical Acoustics (CA) approach, i.e., absorption coefficients and plane wave propagation. However, summation was done in terms of sound pressure instead of energy.

Figure 3 Left, shows the RT spectra of the individual modal components (thin curves) along with their envelope (thick curve) for the room with WPGA (red) and CA (blue). Figure 3 Right, compares the TF of the two cases.

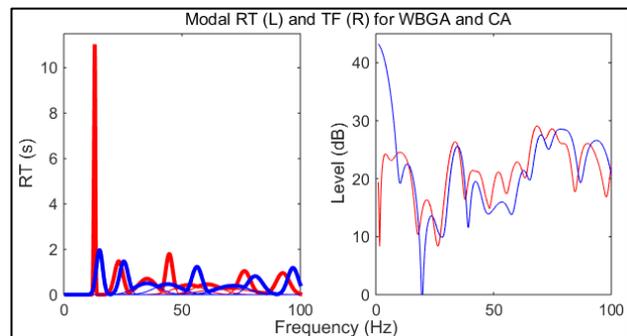


Figure 3. Comparison of the RT spectra of the envelopes and individual modal components of the cuboid room, based on WPGA (red) and CA (blue). Comparison of the levels of the TF, WPGA (red) and CA (blue).

4.3. Simulation of a real room

In order to investigate the methodology on an arbitrary shaped room (whose modal parameters are impossible to calculate analytically), the model of a studio with non-parallel surfaces was used. The room volume is 391m³ while its surface area is 305m².

The geometry of the model is shown in Figure 4, while the absorption area of the materials and Eyring/Sabine RTs are shown in Figures 5.

The orders of reflection were set to 4, while 1 order of diffraction and 1 order of reflections between diffractions were also included. In this case Fresnel correction was used for finite sized surfaces (not applied in the cuboid rooms). The calculations were WBGA and the frequency range spanned from 20 to 150 Hz at 0.1 Hz intervals. Tolerance levels were kept at 1 % for both the natural frequency and loss factor, and the iteration number was changed to 25.

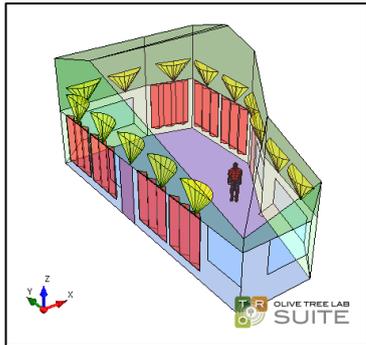


Figure 4. Geometry of the studio under investigation.

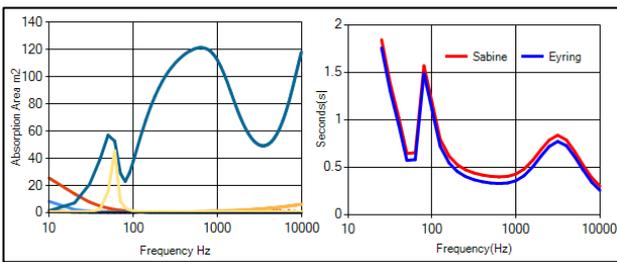


Figure 5. (L) Absorption area of the materials of the studio under investigation. (R) Eyring/Sabine RTs.

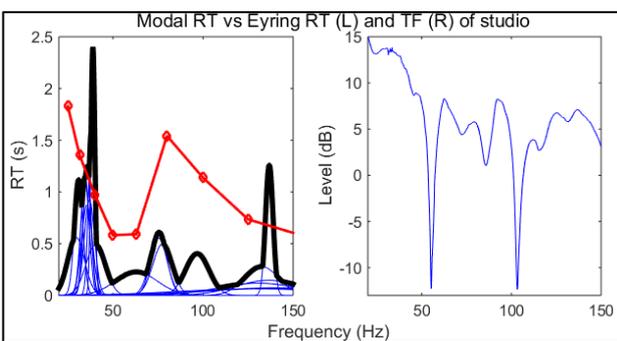


Figure 6. (L) RT spectra of the envelope (thick black curve) and individual components (thin blue curves) vs the Eyring RT (red curve) of the studio. (R) TF of the studio.

Figure 6 on the left shows the RT spectra of the individual modal components (thin blue curve) along with their envelope (thick black curve) vs the Eyring RT (red curve) of the studio while Figure 6 on the right shows the transfer function. Note that

modal RT and TF applies to a particular source receiver position only. Figure 7, for clarity purposes, shows the RT spectra of the individual modal components.

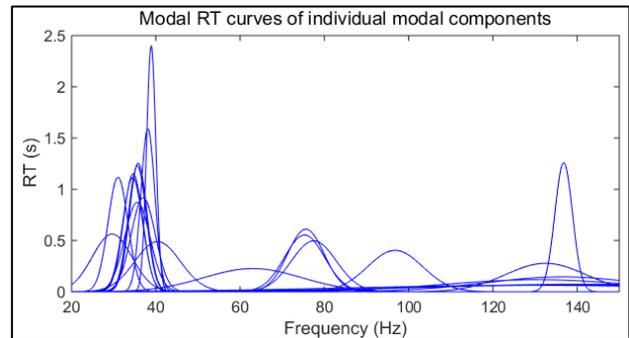


Figure 7. Studio individual resonant frequencies modal RT.

5. Discussion

The effort in this paper is to demonstrate a new approach in calculating modal RT. In essence one needs a TF, modal analysis for extracting room damping and resonant frequencies based on which, modal RT may be calculated and plotted according to the FWHM of each resonant frequency. Eventually the overall modal RT is given by the envelope of individual modal components against frequency. As expected, this is quite challenging, especially for the first two stages.

Comparing the cuboid rooms RT and TF results, one can tell that many of the natural frequencies are missing, however, this no fault of the described methodology, but limitations due to computational power when using the image source method and the low order of reflections used in the particular examples. Lam [4] has shown using WBGA, excellent TF match between measurements and simulations. Longer computations would have revealed better TFs, however, this was not the objective of this paper, but rather to demonstrate the principles of the proposed methodology.

The LSCE method involves trial and error effort to reach converging answers. Depending on the amount of damping present in the room, sound absorption, one needs to decide between frequency resolution extraction and iteration orders. Nevertheless, this a very promising method to be applied in room acoustics.

From the results, the first observation is that in high resolution analysis, there is correlation between low frequency (LF) range TF and RT shapes, something

usually lacking in 1/1 or 1/3 octave band TF and RT plots. Therefore, it is informative to visualize LF range RT levels in terms of modal analysis. Furthermore, this pictorial analysis is more likely to couple perception with calculations.

From Figure 1, one may observe that as sound absorption increases the TF and RT peaks lower. At the same time there is a shift in frequency, to the left, which is the result of using extended impedance calculations which account for angle dependent reflections and phase changes within the surface material. Figure 2 shows that the use of classical acoustics, which ignores phase changes in materials, could never reproduce what sound measurements show. In the case of the studio, the number of dominant resonant modes revealed is much greater than in the cuboid rooms. Such an RT analysis is very useful in fine tuning the acoustics of critical spaces such a recording studio.

6. CONCLUSIONS

The focus of this paper is to demonstrate that room modal analysis reveals the true workings of sound in rooms. Wave Based Geometrical Acoustics has the power of carrying out modal analysis. In conjunction with room modal parameter estimation techniques and as technology progresses, eventually the standard approach would use room modal analysis in all the useful frequency range, thus replacing the current approach of classical acoustics, its assumptions and limitations.

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